

Double Electromagnetically Induced Transparency in a Tripod-type Atom System

Xiao Li, Yu Liu, Hong Guo*

Abstract—The electromagnetically induced transparency (EIT) phenomenon in a four level atomic system with tripod configuration is studied. The results show that this configuration is equivalent to the combination of two single three-level Λ configurations, which, under certain conditions, results in the so-called double-EIT (DEIT) phenomenon. The properties of the double transparency windows for DEIT are discussed in detail and the possible experimental scheme is proposed.

I. INTRODUCTION

ELECTROMAGNETICALLY induced transparency (EIT), as one of the quantum coherence and interference (QCI) phenomena, is an important effect induced by the interaction between laser beams and atom ensembles under two-photon resonance condition [1]-[2]. EIT in three-level systems, such as the typical Λ -type system, has been extensively studied both theoretically and experimentally [3]. Its direct results, such as subluminal [4] and superluminal [5] light propagations, have already been demonstrated experimentally. After that, with the model of dark state polariton [6]-[9], the EIT-based light storage is theoretically proposed and then experimentally realized [10]-[11].

For some types of four-level atomic configurations, similar analysis also shows some interesting phenomena, such as the interchange between subluminal and superluminal propagation [12] and double EIT phenomenon [13]. However, for most of the four-level systems, there is hardly ideal dark states and so brings serious limitations to their applications. Fortunately, as we will show in this paper, the four-level tripod-type atom system does yield an ideal dark state, which may greatly improve the coherence performances of current four-level atomic systems.

In Fig. 1, we show the schematic setup of a four-level tripod-type atomic system. A linearly polarized (π) light is served as the coupling light while the probing light is composed of left and right polarized components σ_{\pm} with the same frequency ω_p and the same amplitude. In this configuration, the probing light is traveling in the direction perpendicular to both the polarization of the coupling light and the direction of the magnetic field, which is used to split the ground level into three Zeeman sub-levels ($m_F = -1, 0, +1$). Thus, this system is equivalent to the combination of two simple Λ -type systems coupled by a common light.

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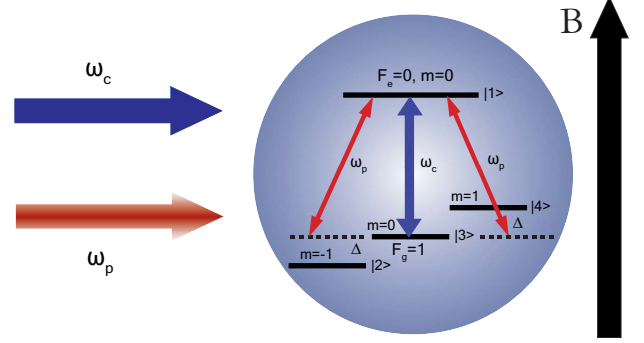


Fig. 1. (color online) The configuration of the tripod-type four-level atom. $|1\rangle$ is the excited state with the damping rates Γ_{12} , Γ_{13} , and Γ_{14} . $|2\rangle$, $|3\rangle$, $|4\rangle$ are the ground states split by an external magnetic field, while Δ is the splitting gap. Levels $|1\rangle$ and $|2\rangle$, $|1\rangle$ and $|4\rangle$ are coupled by the left and right polarized components of a weak probing light Ω_p , respectively, while levels $|1\rangle$ and $|3\rangle$ are coupled by a strong light Ω_c .

It has been shown [15] that the coherence situation of this configuration can be altered by adjusting the the Rabi frequency of the coupling light Ω_c and the Zeeman splitting Δ . The double EIT phenomenon can thus be modulated by the Rabi frequency Ω_c and thereby the frequency differences between the coupling and the probing light, i.e., $\delta = \omega_p - \omega_c$. However, there are following issues left to be explored: 1) a systematic discussion of the role of each parameters in this configuration, 2) the impact of the non-radiative damping on the coherence of the system, and 3) the conditions required for experimental realization. All of the above issues will be discussed in this paper. We will first give numerical solutions to the equation of motion for a tripod-type four-level atomic system and discuss its absorption and dispersion properties. Then, explicit expressions will be given to show that the four-level tripod-type system is indeed a combination of two three-level Λ -type systems. Following that, we will show how to make this system an optimal EIT system by adjusting related parameters and then, we will discuss the experimental possibilities.

II. ABSORPTION AND DISPERSION FOR A TRIPOD-TYPE FOUR-LEVEL ATOMIC SYSTEM

In the four-level atomic system shown in Fig. 1, the Hamiltonian under the rotating wave frame can be written as

$$\begin{aligned} \hat{H} = & \hbar(\omega_p - \omega_{12})|2\rangle\langle 2| + \hbar(\omega_c - \omega_{13})|3\rangle\langle 3| \\ & + \hbar(\omega_p - \omega_{14})|4\rangle\langle 4| \\ & - \frac{\hbar}{2} (\Omega_p|1\rangle\langle 2| + \Omega_c|1\rangle\langle 3| + \Omega_p|1\rangle\langle 4| + \text{H.c.}), \quad (1) \end{aligned}$$

where Ω_p and Ω_c are the Rabi frequencies and $\omega_{ij} = \omega_i - \omega_j$ is the central frequency between Zeeman sub-levels $|i\rangle$ and $|j\rangle$.

A general expression of the eigenstates of the Hamiltonian is very complicated. However, when the Zeeman splitting is equal to the frequency difference between the two lights, an ideal dark state, whose eigenvalue is zero, emerges, with the expression:

$$|\Psi^0\rangle = -|2\rangle + |4\rangle, \quad (2)$$

while the other three eigenstates are

$$|\Psi_i\rangle = -\frac{2\lambda_i}{\hbar\Omega_p}|1\rangle + |2\rangle - 2\frac{\hbar^2\Omega_p^2 - 2\lambda_i^2}{\hbar^2\Omega_c\Omega_p}|3\rangle + |4\rangle,$$

with eigenvalues λ_i ($i = 1, 2, 3$) satisfying

$$4\lambda^3 - 8\hbar\delta_c\lambda^2 - \hbar^2(\Omega_c^2 + 2\Omega_p^2)\lambda + 4\hbar^3\delta_c\Omega_p^2 = 0.$$

Next we will take into account the decays of the atomic levels due to radiative and non-radiative dampings. We base our discussion on the steady state solution of the density matrix equation. For simplicity, we assume that Ω_p and Ω_c are real and the coupling light interacting with the states $|1\rangle$ and $|3\rangle$ is resonant ($\omega_c = \omega_{13}$). Suppose that all radiative damping rates are equal, i.e., $\Gamma_{12} = \Gamma_{13} = \Gamma_{14} = \beta\gamma$, where γ is atomic spontaneous emission rate, and so are the nonradiative damping rates, i.e., $\Gamma_{ij} = \alpha\gamma$, where $i, j = 2, 3, 4$ ($i \neq j$). Also, we set $\omega_{32} = \omega_{43} = \Delta\gamma$, $\Omega_p = g_p\gamma$, $\Omega_c = g_c\gamma$, and $\delta = \omega_p - \omega_c = \delta_c\gamma$ for normalization. Here, Γ_{ij} represents the damping rate from state $|i\rangle$ to $|j\rangle$, $\omega_{ij} = \omega_i - \omega_j$, and δ is the frequency difference between the coupling and probing lights. Under the above assumptions, we can get the steady-state solution for these equations.

To see the absorption and dispersion characteristics of the probing light, we plot the steady state solution of $(\rho_{12} + \rho_{14})/g_p$ in Fig. 2, where $\Delta = 5.0$, $\alpha = 0.001$, and $\beta = 0.666$. From the figure, one finds that this four-level atomic system has double transparency windows, each of which is the typical result of a Λ -type system. The concrete discussion on this result will be given in the next section.

The following analytical results will show that the tripod scheme can, under some approximations, be viewed as a combination of two Λ -type schemes. Firstly, the expression for $(\rho_{12} + \rho_{14})/g_p$, which represents the absorption and dispersion characteristics of the tripod system, can further be simplified if we neglect the terms with higher orders of g_p , as shown below:

$$h = \frac{\rho_{12} + \rho_{14}}{g_p} = \frac{i\beta}{9\alpha\beta + 4(2\alpha + \beta)g_c^2}(h_l + h_r), \quad (3)$$

where

$$h_l = \frac{3\alpha(2\alpha + i\Delta - i\delta_c) + g_c^2(\alpha + 2i\Delta - 2i\delta_c)}{g_c^2 - (i - \Delta + \delta_c)(2i\alpha - \Delta + \delta_c)},$$

$$h_r = \frac{3\alpha(2\alpha - i\Delta - i\delta_c) + g_c^2(\alpha - 2i\Delta - 2i\delta_c)}{g_c^2 - (i + \Delta + \delta_c)(2i\alpha + \Delta + \delta_c)}.$$

For simplicity, the damping rates between the ground states are ignored at the moment and the validity of this approximation will be discussed at the end of the paper. Then, one

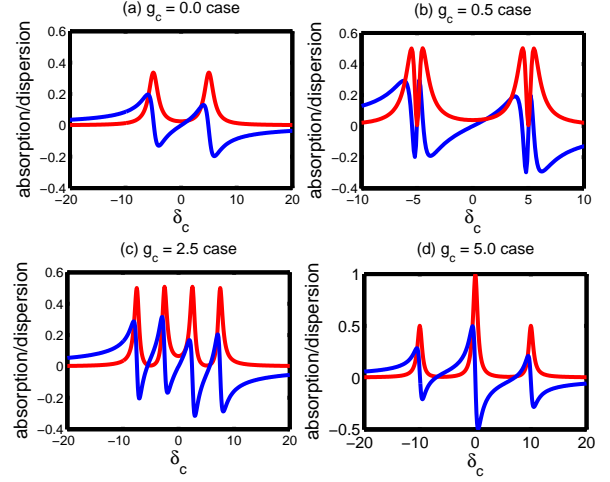


Fig. 2. (color online) Absorption and dispersion of the four-level tripod configuration. Here the blue curve represents the dispersion, while the red curve represents absorption. The parameters are $\Delta = 5.0$, $\alpha = 0.001$, and $\beta = 0.666$. The differences result from the different values of g_c .

has:

$$h = \frac{\rho_{12} + \rho_{14}}{g_p} = \frac{1}{2}[h_0(\delta_c - \Delta) + h_0(\delta_c + \Delta)], \quad (4)$$

with

$$h_0(x) = \frac{x}{g_c^2 - x^2 + ix}. \quad (5)$$

It is clear from Eq. (4) that the system is equivalent to a combination of two symmetric parts. Besides, some important properties of $h_0(x)$ is worth discussing here. We note that, according to Eq. (5), its imaginary part reads

$$\text{Im}[h_0(x)] = \frac{x^2}{(g_c^2 - x^2)^2 + x^2}, \quad (6)$$

One can easily verify that this function has a minimum of zero at $x = 0$. Also, it has two peaks at $x = \pm g_c$, respectively. Therefore, the distance between the two peaks is $2g_c$. These conclusions will be useful in our later discussion. We now go on to show that each part of Eq. (4) is exactly the same as that of a Λ -type scheme.

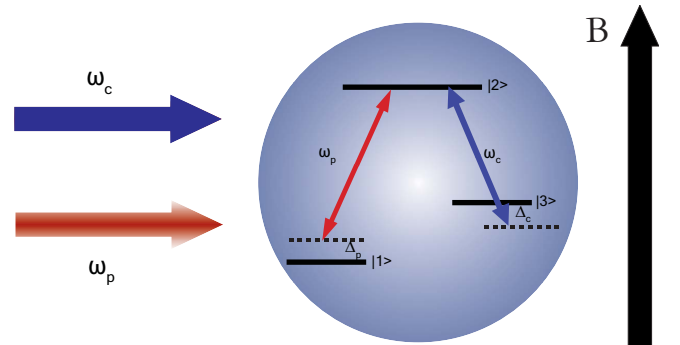


Fig. 3. (color online) The configuration of the three level Λ -type atom. $|1\rangle$ and $|3\rangle$ are two ground state sub-levels while $|2\rangle$ is the excited state with the damping rates Γ_{21} and Γ_{23} . Ω_p and Ω_c are the Rabi frequencies of the probing and the coupling light, respectively. Δ is the splitting caused by Zeeman effect.

The standard three level Λ -type scheme is shown in Fig. 3. Following the same approach as mentioned above, we first give the steady-state solution of ρ_{21} , the real and imaginary part of which represent the dispersion and the absorption properties of the system, respectively:

$$h_1 = -A/B,$$

where

$$A = g_c^2 [g_c^2 + g_p^2 - (\Delta - \delta_c) (i + \Delta - \delta_c)] (\Delta - \delta_c),$$

$$B = g_c^2 \left[3g_p^4 + (\Delta - \delta_c)^2 (1 + \Delta^2 - 2\Delta\delta_c + \delta_c^2 + 4g_p^2) \right] + g_c^4 \left[3g_p^2 - 2(\Delta - \delta_c)^2 \right] + g_c^6 + g_p^2 \left[g_p^4 + (\Delta - \delta_c)^2 \right].$$

Since the probing light is much weaker than the coupling light, the above expression can be simplified as:

$$h_1 = \frac{\delta_c - \Delta}{g_c^2 - (\delta_c - \Delta) (i + \delta_c - \Delta)}, \quad (7)$$

which is exactly the same as $h_0(\delta_c - \Delta)$, the first term of the four-level result [see Eq. (4)]. Here, $\Omega_c = g_c\gamma$ is the Rabi frequency of the coupling light, $\Delta\gamma = \omega_{31}$ denotes the Zeeman splitting of the ground level, and $\delta_c\gamma = \omega_p - \omega_c$ is the frequency difference between the coupling and the probing lights. Fig. 4 shows the dispersion and absorption properties of this three-level system, which further confirms our conclusion that the tripod-type atomic system can, under the weak probing light assumption, be viewed as the combination of two independent Λ -type systems.

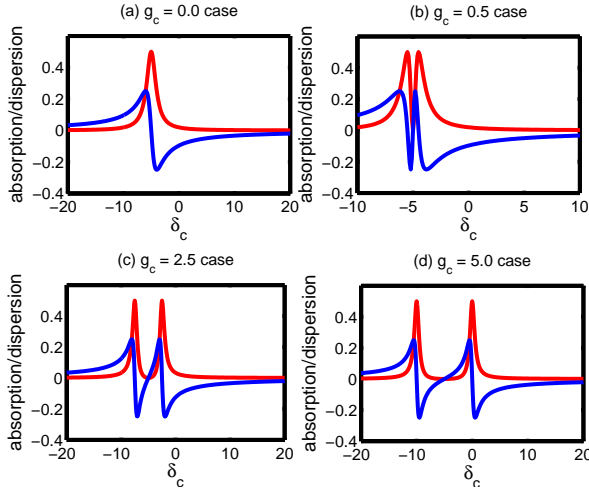


Fig. 4. (color online) Dispersion and absorption of the three-level Λ configuration. Likewise, the blue curve represents the dispersion, while the red curve represents the absorption. The related parameters are $\Delta = 5.0$, $\alpha = 0.001$, and $\beta = 0.666$. Ω_c is used to modulate the system. From the figures, one recognizes that the tripod-type is equivalent to the combination of two Λ systems.

III. THE IMPACT OF THE COUPLING LIGHT AND THE MAGNETIC FIELD

In the previous section, we have reached the conclusion that the tripod-type atomic system is equivalent to the combination of two Λ -type ones. Here we want to figure out in what way the two Λ -type schemes constitute the two transparency windows.

Recalling the properties of $h_0(x)$, we can conclude that $h_0(\delta_c - \Delta)$ has its minimum at $\delta_c - \Delta = 0$, with the two peaks at $\delta_c - \Delta = \pm g_c$, respectively. That is to say, the right transparency window will have a central frequency of $\delta_c = \Delta$ and a width $2g_c$. This conclusion is also valid for the left transparency window, i.e., it is centered at $\delta_c = -\Delta$ with the width $2g_c$. The above discussion gives us some hint on how to construct the two transparency windows. If we choose to fix the external magnetic field and scan the frequency of the probing light, the two transparency windows are expected to emerge when the frequency difference between the probing light and the pumping light satisfies $\delta_c = \pm\Delta$, respectively.

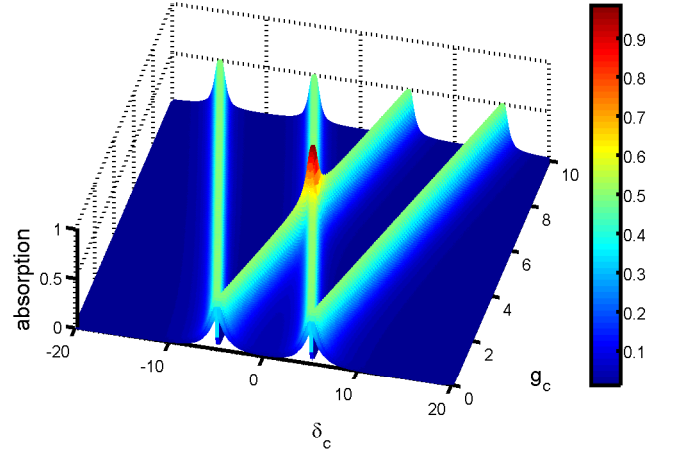


Fig. 5. (color online) The impact of the coupling light. Here we keep the Zeeman splitting $\Delta = 5$ while the Rabi frequency of the coupling light g_c is modulated. The color represents the magnitude of the absorption strength, as shown in the bar. The absorption and dispersion under four typical values of g_c have been shown in Fig. 2.

The Rabi frequency of the coupling light $\Omega_c = g_c\gamma$ is of interest here. We want to show that it should not be too large in order to obtain satisfied transparency windows. The first reason is very apparent, since a narrow transparency window is always desirable for obtaining slow light and thereby the light storage. The second reason is not so close at hand: We will show this point in Fig. 5, where the Zeeman splitting is kept constant $\Delta = 5$. One can see that with the increase of g_c , the width of the two transparency windows becomes ever larger. When $g_c > \Delta$, however, the two windows begin to have overlap, which significantly alters the absorption properties of the system. In this case, the two windows will be centered at $\pm g_c$, with the width of 2Δ . Given the above two reasons, we conclude that when using the tripod-type scheme, the Rabi frequency of the coupling light is preferable to be less than the Zeeman splitting. There is another interesting result in Fig. 5. When g_c equals to Δ , we observe the so-called EIA phenomenon[16], in which the absorption of the system is doubled at $\delta_c = 0$.

Next, we fix the frequency of the probing light ω_p , while scanning the Zeeman splitting Δ . The result is shown in Fig. 6. It can be seen that when the magnetic field is turned off, there is full absorption at $\delta_c = g_c$ [see Fig. 7(a)]. This is understandable, since when $\Delta = 0$ this system is degenerated

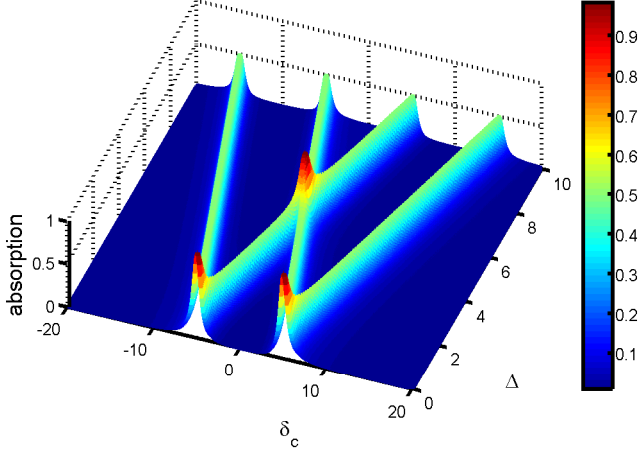


Fig. 6. (color online) The impact of the magnetic field. Here we keep the Rabi frequency of the coupling light to be $g_c = 5$ while the Zeeman splitting Δ is modulated. The color represents the magnitude of the absorption, as shown in the bar. Fig. 7 shows the absorption and dispersion under four typical values of Δ .

as a two-level configuration, and thus there exists only full absorption. After applying the magnetic field, the absorption peaks are split into two halves, and the transparency windows are established [Fig. 7 (b)-(d)]. We want to further mention that similar phenomena, such as the EIA, and the overlap of two transparency windows will happen in the scan of the Zeeman splitting.

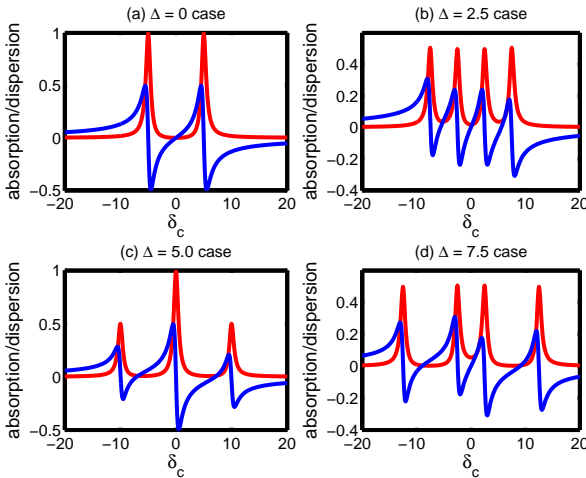


Fig. 7. (color online) The absorption and dispersion when $\Delta = 0.0, 2.5, 5.0$ and 7.5 , respectively. Here, the blue curve represents the dispersion, while the red curve represents the absorption. The related parameters are $g_c = 5.0$, $\alpha = 0.001$, and $\beta = 0.666$.

IV. DISCUSSION AND CONCLUSION

The experimental realization using this four-level system has the following concerns: the real atomic system, the damping rates for each level, the magnitude of the magnetic field and the influence of atomic collisions.

There have been many atomic systems that can satisfy our requirements, such as the $4f^6 6s^2 \ ^7F_1 \leftrightarrow 4f^6 6s 5p \ ^6D_0$

transition of Sm [17]-[18], the $2p^5 3s \ ^3P_1 \leftrightarrow 2p^5 3p \ ^3P_0$ transition of Ne [19], and also the $5s^1 \ ^2S_{1/2} \leftrightarrow 5p^1 \ ^2P_{3/2}$ transition of ^{87}Rb (D2 line). Since the laser for the wavelength of $\lambda = 780 \text{ nm}$ is commercially very popular, this adoption may be more readily for applications.

Another important issue is the impact of damping rates. Since the dark state of this system [see Eq. (2)] does not contain the upper level $|1\rangle$, it is immune to radiative damping. Therefore, this four-level tripod-type system is much more rigid, in fighting with decoherence due to radiative dampings, than a four-level N -type atomic system [20]. Then, a question naturally arises: is the non-radiative damping, i.e., the dephasing from collisions, an urgent problem in the current configuration? This question is also crucial for the validity of the approximation used in Eq. (4). Fig. 8 shows the absorption of the tripod-type system under different non-radiative damping rates, where α is the ratio of non-radiative to radiative damping rate, as we defined before. Fortunately, We can see that the general properties, i.e., the central frequency and width of the transparency windows, have been kept. Only the maximal absorption is reduced, which is not a crucial problem, since it still yields a good contrast. Moreover, given that non-radiative damping is much smaller than the radiative damping, its limitations on operation time is significantly less. Therefore, we can conclude that the non-radiative damping rates, though slightly changes the absorption of this system, is not a crucial problem, and our previous approximation is valid.

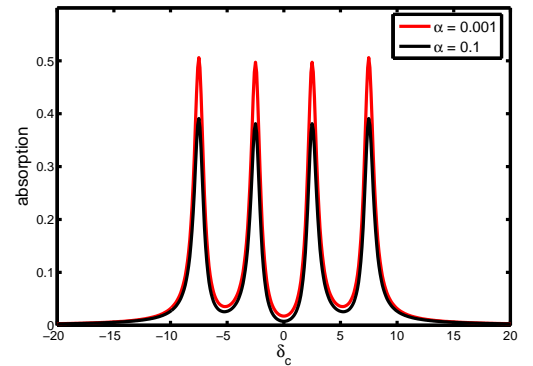


Fig. 8. (color online) The impact of non-radiative damping. The red and black curve corresponds to a non-radiative damping rate of $\alpha = 0.001, 0.1$, respectively. Here, the related parameters are: $\beta = 0.666$, $\Delta = 2.5$, and $g_p = 5.0$.

Next, we briefly discuss the requirement of the magnetic field. Since the normal Zeeman splitting and the external magnetic field B has the relation: $\Delta = g\mu_b B/\hbar$, where g is the Landé factor and μ_b is the Bohr magneton, then, if we need to realize a frequency shift in ^{87}Rb of, say, 10 MHz, a magnetic field of about 7 Gauss is needed and can be easily fulfilled.

In this paper, we discuss in detail the properties of a four-level tripod-type EIT system. We first show that this system has two transparency windows, then analysis are made to demonstrate that this configuration is actually a combination of two three-level Λ -type EIT systems. Following that, we focus

on the discussions of the properties of the transparency windows. We show that the profile of the transparency windows rely strongly on the relative magnitude of the Rabi frequency of the coupling light and the Zeeman splitting. In the end, issues related to the experimental realization of this scheme are discussed. As a whole, it is pointed that the four-level tripod-type configuration yields an ideal dark state, which is very rare in four-level systems and is very beneficial for obtaining EIT in four-level systems. Besides this, it should also be noted that the ideal double dark states and double EIT windows have potential applications in the light storage for, at least, two frequencies of light and thus, could be connected with all-optical communication, together with the usage of wave division multiplexing (WDM) techniques.

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